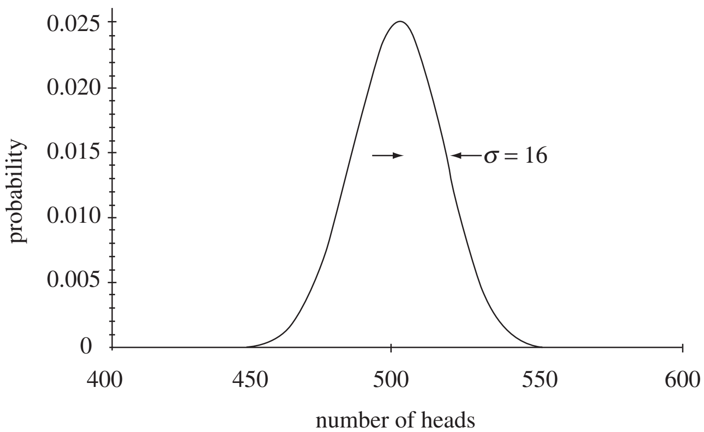

Figure 3.1 Probabilities for various numbers of heads when 1000 coins are flipped. The distribution peaks at 500 heads, for which the probability is 0.0252. As you can see, the chances of getting less than 450 or more than 550 heads are negligible.

It would be less work, less confusing, and nearly equally informative if we could just calculate the following two numbers:

- the average number of coins that would land heads if the coin-flip experiment were repeated many times;
- some measure of the fluctuations we could expect around this value.

In the above case of 1000 flipped coins, for example, it is extremely likely that the number of heads will fall between 450 and 550 (Figure 3.1). But the probability of getting *exactly* 500 heads is only 0.0252.

A.1 Mean value and standard deviation

We now investigate how to calculate mean values and characteristic fluctuations for any system. We will imagine that we have a large number of such systems, which have been prepared in the same way (an "ensemble"). For example, we might have many systems of 1000 flipped coins. Equivalently, we might flip the same set of 1000 coins many different times.

For large numbers of identically prepared systems having N elements each, the average number of elements per system that satisfy a criterion is given by

$$\overline{n} = pN, \tag{3.1}$$

where p is the probability for any given element to satisfy the criterion. We can think of this as the definition of the probability p: it is the fraction of the total number of elements that satisfy the criterion. Alternatively, this relationship can be derived from the definition of mean values 2.1.

The average fluctuation of n about its mean value must be zero, because the definition of the mean value guarantees that the positive fluctuations cancel the negative ones. But the squares of the fluctuations are all positive numbers. So if