solving FDEs and fractional integro–differential equations (FIDEs) is the operational matrix. For example Lakestani et al. (2012) discussed the operational matrix for FDEs by using linear B-spline functions. Also, Saadatmandi and Dehghan (2010) suggested an operational matrix for FDEs by generalizing the Legendre operational matrix. Li and Zhao (2010) and Li and Sun (2011) used the Haar wavelet operational matrix and block pulse operational matrix for FDEs, and Kilicman and Al Zhour (2007) discussed a method for fractional calculus based on Kronecker operational matrices. Moreover, Mokhtary and Ghoreishi (2011) studied the Legendre spectral Tau matrix method for nonlinear fractional integro–differential equations. We refer the interested reader to Agrawal and Baleanu (2007); Muslih and Baleanu (2007); Muslih et al. (2007); Rabei et al. (2007a, 2007b); Baleanu and Trujillo (2008); Dehghan et al. (2010a, 2010b); Esmaeili and Shamsi (2011); Saadatmandi and Dehghan (2011); Yousefi et al. (2011a, 2011b); Saadatmandi et al. (2012) for some advances in the subject of fractional differential equations.

In the present paper, we first introduce the fractional orthogonal Jacobi functions then we obtain the explicit form of the fractional derivative operational matrix for these functions. Finally we apply it to solving FDEs and FIDEs (for the arbitrary parameters α and β). Note that in this way we assume that the fractional derivative is defined as the left-side Caputo derivative.

Definition 1: The Caputo fractional derivative of $f(x)$ of order $\lambda > 0$ with $a \geq 0$ is defined as

$$\left(D_a^\lambda f\right)(x) = \frac{1}{\Gamma(\beta - \lambda)} \int_a^x \frac{f^{(\beta)}(t)}{(x-t)^{\lambda+1-\beta}} \, \mathrm{d}t$$

for $\beta - 1 < \lambda \leq \beta$, $\beta \in \mathbb{N}$, $x \geq a$. For example, if $f(x) = (x-a)^k$ for $k \geq 0$ we have (Diethelm, 2004)

$$D_a^\lambda f(x) = \begin{cases} 0, & \text{if } k \in \{0, 1, 2, \ldots, \beta - 1\}, \\ \dfrac{\Gamma(k+1)}{\Gamma(k+1-\lambda)}(x-a)^{k-\lambda} & \text{if } k \in \mathbb{N} \text{ and } k \geq \beta \text{ or } k \notin \mathbb{N} \text{ and } k > \beta - 1 \end{cases}$$

where $\beta = \lceil \lambda \rceil$, $\lambda \geq 0$. We use the ceiling function $\lceil \lambda \rceil$ to denote the smallest integer greater than or equal to λ. Furthermore the Caputo fractional differentiation is a linear operation

$$D_a^\lambda(n_1 f_1(x) + n_2 f_2(x)) = n_1 D_a^\lambda f_1(x) + n_2 D_a^\lambda f_2(x) \quad (1)$$

where n_1, n_2 are constants, and we have

$$D_a^\lambda C = 0, \quad \text{for any constant } C \quad (2)$$

For more properties of Caputo fractional derivatives, the reader should see Miller and Ross (1993); Diethelm (2004); Kilbas et al. (2006); Baleanu et al. (2012).

This paper is organized as follows, Sections 1 and 2 present the preliminary definitions and the main theorems. Section 3 presents a method for obtaining the operational matrix for fractional orthogonal Jacobi functions. Applications of the operational matrix are given in Section 4. Section 5 includes some illustrative examples. Section 6 completes this paper with concluding remarks.

1.1. The fractional Taylor series

In the classical Taylor's formula, we approximate the function $f(x)$ via polynomials $p_n(x)$, $n \in \mathbb{N}$, with this purpose if the $(n+1)$st-order derivative, $f^{(n+1)}(x)$, exists for all x in an interval containing c and x, and if $p_n(x)$ is the nth-order Taylor polynomial for $f(x)$ about c, i.e.

$$f(x) \simeq p_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(c)}{k!}(x-c)^k$$

then the error $E_{n,c}(x) = f(x) - p_n(x)$ in the approximation $f(x) \simeq p_n(x)$ is given by

$$E_{n,c}(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-c)^{n+1}$$

where ξ is some number between c and x. The resulting formula

$$f(x) = \sum_{k=0}^{n} \frac{f^{(k)}(c)}{k!}(x-c)^k + E_{n,c}(x) \quad (3)$$