CHAPTER

1 Introduction

Compressive sensing is a radical new way of sampling signals at a sub-Nyquist rate. The Shannon/Nyquist sampling theorem states that an analogue signal can be reconstructed perfectly from its samples, if it was sampled at a rate at least twice the highest frequency present in the signal (Nyquist 1928; Shannon 1949). This rate is known as the *Nyquist* or *Shannon* rate of that signal, and for many signals, such as audio or images, the Nyquist rate can be very high. This may result in acquiring a very large number of samples, which must be compressed in order to store or transmit them, as well as placing a high requirement on the equipment needed to sample the signal. *Compressive Sensing* (also referred to as compressed sensing or CS) is a recently introduced method that can reduce the number of measurements required; in some ways it can be regarded as automatically compressing the signal. Compressive sensing is a technique that enables us to fully reconstruct particular classes of signals if the original signal is sampled at a rate well *below* the Nyquist rate.

In particular, compressive sensing works with sparse signals. In many applications the signal of interest is primarily zero, that is, the signal has a representation in some pre-determined basis in which most of the coefficients are 0. Traditionally measurement techniques heavily over sample the signal. Consider the scenario where we randomly draw samples from a sparse signal, so the probability of sampling at an "interesting" data point is equal to the sparsity fraction. Compressive sensing avoids excessive oversampling by using linear sampling operators – a combination of sampling and compression, giving rise to its name.

One of the original breakthroughs in compressive sensing by Candès, Romberg, Tao in (Candès, Romberg, and Tao 2006a; Candès, Romberg, and Tao 2006b; Candès and Tao 2006; Candès and Tao 2005) and Donoho in (Donoho 2006) was to show that linear programming methods can be used to efficiently reconstruct the data signal with high accuracy. Since then many alternative methods have been proposed as a faster or superior (terms of reconstruction rate) alternative to these linear programming algorithms. One approach is to use matching pursuit techniques, originally proposed in (Mallat and Zhang 1993), variations have been proposed such as OMP or orthogonal matching pursuit (Tropp and Gilbert 2007), Stagewise orthogonal matching pur-