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$$\begin{array}{ccc} & \mathcal{U}_\alpha & \\ i^\alpha \nearrow & & \searrow I^\alpha \\ \Sigma_g & & \mathcal{U}_\alpha \cup_\Sigma \mathcal{U}_\beta \\ i^\beta \searrow & & \nearrow I^\beta \\ & \mathcal{U}_\beta & \end{array}$$
$$\begin{array}{ccccc} & & BSpin_n^{\mathbb{C}} & & \\ \tilde{f} \nearrow & & \downarrow & & \searrow \alpha \\ M & & & & B(SO_n \times U(1)) \\ f \searrow & & \swarrow & & \\ & BSO(n) & & & \end{array}$$

(1)

$$\begin{array}{ccccc}
\mathbb{Z} & \subset & U(1) & \longrightarrow & U(1) \\
\downarrow \approx & & \downarrow & & \downarrow \\
\mathbb{Z} & \subset & Spin_n^{\mathbb{C}} & \longrightarrow & SO_n \times U(1) \\
& & \downarrow & & \downarrow \\
& & SO_n & \approx & SO_n
\end{array} \tag{2}$$