

generally, $(x - a, y - b)$ is prime for any $(a, b) \in \mathbb{C}^2$. Also, if $f(x, y)$ is an irreducible polynomial (e.g. $y - x - 2$, $y - x^2$ or $y^2 - x^3$) then $(f(x, y))$ is prime.

We can now attempt to draw a picture of this space. The maximal primes correspond to the old-fashioned points in \mathbb{C}^2 : $[(x - a, y - b)]$ corresponds to $(a, b) \in \mathbb{C}^2$. We now have to visualize the bonus points. $[(0)]$ somehow lives behind all of the old-fashioned points; it is somewhere on the plane, but nowhere in particular. So for example, it does not lie on the parabola $y = x^2$. The point $[(y - x^2)]$ lies on the parabola $y = x^2$, but nowhere in particular on it (Figure 4.3). You can see from this picture that we already want to think about dimension. The primes $(x - a, y - b)$ are somehow of dimension 0, the primes $(f(x, y))$ are of dimension 1, and (0) is somehow of dimension 2. (All of our dimensions here are complex or algebraic dimensions. The complex plane \mathbb{C}^2 has real dimension 4, but complex dimension 2. Complex dimensions are in general half of real dimensions.) We won't define dimension precisely until later, but you should feel free to keep it in mind before then.

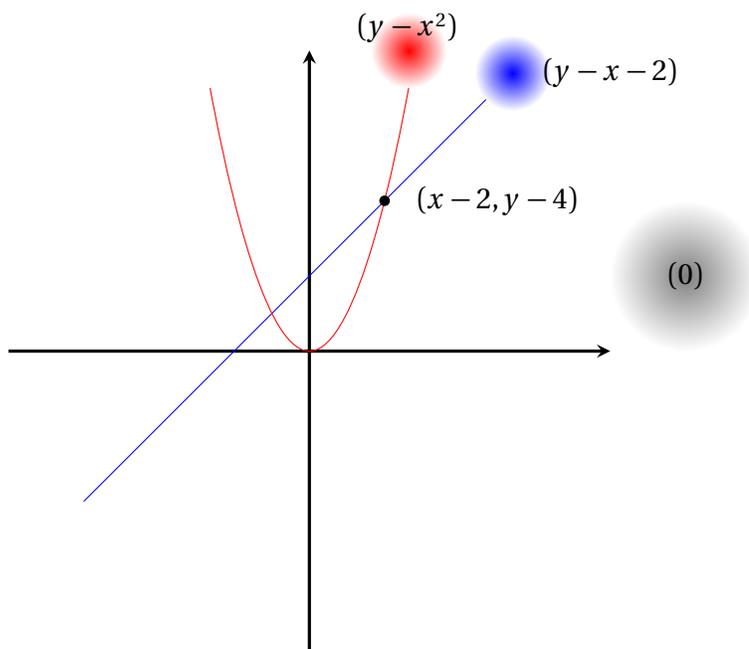


Figure 4.3: Affine plane $\mathbb{A}_{\mathbb{C}}^2$