$ARIMA$
(d)
$ARIMA$
$ARFIMA$
$(ARFIMA)$
$\phi(L)(1-L)^d x_i = \theta(L)\varepsilon_t$
(1)
$ARMA$
d
$(I(0))$
$(I(1))$
$(I(d))$
d
$ARIMA$
$ARFIMA$
$ARFIMA$
X_t
ρ_j
j

$$\lim_{n\to\infty}\sum_{i=-n}^{n} \mid \rho_i \mid = \infty$$

(2)
$\rho_i \sim$
$C \mid$
$i \mid^{2d-1}$
$i \to$
$\infty, C \neq$
$0, 0 <$
$d <$
12
$d <$
0
$\sum_{-\infty}^{\infty} |\rho(k)| <$
∞
$0 <$
$d <$
0.5
$\sum_{-\infty}^{\infty} |\rho(k)| =$
∞
$ARMA(p,q)$
p
q
$\{x_t, t =$
$0, \pm 1, \pm 2, \cdots\}$
$ARIMA(0,d,0)$
$d \in$
$(-0.5, 0.5)$
$\{x_t\}$
$\nabla^d x_t =$
z_t
$\{z_t\} WN(0,\sigma^2)$
$\{x_t\}$
$\mathbf{X}_t =$
$(X_{1,t}, \cdots, X_{r,t})', t =$
$1, 2, \cdots, T$
$diag(\nabla^d)\mathbf{X}_t = \mathbf{Z}_t$
(3)
$\mathbf{X}_t = diag(\nabla^{-d})\mathbf{Z}_t$
(4)
$\mathbf{X}_t$
r
$diag(\nabla^d)$

$$diag(\nabla^d) = \begin{bmatrix} \nabla^{d_1} & 0 & \cdots & 0 \\ 0 & \nabla^{d_2} & \cdots & 0 \\ & & & \\ 0 & 0 & \ldots & \nabla^{d_r} \end{bmatrix}$$

$\nabla =$
$1-$
B
$\mathbf{Z}_t =$
$(Z_{1,t}, \cdots, Z_{r,t})'$
Σ
$\Delta =$
$1-$
B
$\nabla x_t = (1-L)x_t = z_t$
(5)
L
z_t
σ^2

$$\nabla^d = (1-L)^d = \sum_{k=0}^{\infty} \binom{d}{k} (-L)^k = 1 - dL - 12d(d-1)L^2 - 16d(d-1)(d-2)L^3 - \cdots$$

(6)
L