- $s \vDash p$ iff $s \in \|p\|$.
- $s \vDash \neg\varphi$ iff $s \nvDash \varphi$.
- $s \vDash \varphi \wedge \psi$ iff both $s \vDash \varphi$ and $s \vDash \psi$.
- $s \vDash \Box_A \varphi$ iff $t \vDash \varphi$ for all t for which $s \xrightarrow{A} t$.

We can make use of the following abbreviations.

- $\varphi \vee \psi \equiv \neg(\neg\varphi \wedge \neg\psi)$.
- $\varphi \rightarrow \psi \equiv \neg\varphi \vee \psi$.
- $\varphi \leftrightarrow \psi \equiv (\varphi \rightarrow \psi) \wedge (\psi \rightarrow \varphi)$.
- $\Diamond_A \varphi \equiv \neg\Box_A \neg\varphi$.

We can use these abbreviations as though they are part of the original language. We distinguish these symbols from symbols of the original language by calling the original *primitive symbols*. We did not include these abbreviations in the original language, as it is often helpful to keep the primitive symbols to a minimum. In particular, fewer primitive symbols result in fewer cases needed in inductive proofs.

Observe that from the semantics of $\Box_A \varphi$, we can determine that $s \vDash \Diamond_A \varphi$ iff there exists a t for which $s \xrightarrow{A} t$ and $t \vDash \varphi$. Thus $\Box_A$ and $\Diamond_A$ are related to $\forall$ and $\exists$ respectively. We can read $\Diamond_A \varphi$ as "A considers φ possible".

2.1.2. Adding Common Knowledge. In the language above, we can express that everyone knows φ using the formula

$$\Box\varphi \equiv \bigwedge\{\Box_A \varphi : A \in \mathcal{A}\}$$

But often it is relevant whether the agents are aware of this mutual knowledge. We can write that not only does everyone know φ, but everyone knows that everyone knows φ:

$$\Box\varphi \wedge \Box\Box\varphi$$